

Closing today: 3.4(part 2)  
Closing Wed: 10.2  
Closing Fri: 3.5(part 1)  
Office Hours: 1:30-3:30 in THO 335

*Entry Task:*

In both cases, find  $dy/dx$ .

### **3.5 Implicit Differentiation**

*Motivation:* Consider the unit circle

$$x^2 + y^2 = 1$$

This equation does NOT define a function. We say it *implicitly* defines more than one function.

In particular, it defines:

$$y = \sqrt{1 - x^2} \quad \text{or} \quad y = -\sqrt{1 - x^2}$$

## *General Notes*

We say  $F(x,y) = 0$  implicitly defines one or more functions  $y = y(x)$ .

If we can think of  $y$  as a function of  $x$ , then we can differentiate  $F(x,y(x)) = 0$ , directly.

**But**, we must appropriately use the chain rule.

## *Inverse Functions:*

We write inverse functions as

$y = f^{-1}(x)$  which is equivalent to

$$f(y) = x .$$

We can implicitly differentiate

$$\frac{d}{dx} [f(y) = x] \Rightarrow f'(y) \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{f'(y)}$$